

# Interacting dark fluid in the universe bounded by event horizon;A non-equilibrium prescription

Subenoy Chakraborty · Atreyee Biswas

Received: date / Accepted: date

**Abstract** A non-equilibrium thermodynamic analysis has been done for the interacting dark fluid in the universe bounded by the event horizon. From observational evidences it is assumed that at present the matter in the universe is dominated by two dark sectors-dark matter and dark energy. The mutual interaction among them results in spontaneous heat flow between the horizon and the fluid system and the thermal equilibrium will no longer hold. In the present work, the dark matter is chosen in the form of dust while the dark energy is chosen as a perfect fluid with constant equation of state in one case and holographic dark energy model is chosen in the other. Finally, validity of the generalized second law of thermodynamics has been examined in both cases.

**Keywords** Dark matter · Dark energy · Interaction · Irreversibility

**PACS** 98.80.cq · 98.80.-k

## 1 Introduction

At present, based on the recent observational evidences (particularly from Type Ia supernovae observations[1]) it is commonly believed that the matter in the universe is dominated by two dark components-dark matter (about 23%) and dark energy (about 73%). Dark matter (DM), the invisible matter without pressure can explain galactic curves and large-scale structure formation while dark energy (DE), an exotic matter

with large negative pressure is responsible for the present accelerating phase of universe. Although there are several proposals for DE candidate, but still the nature of dark energy is completely unknown.

However, the most common choice for DE candidate namely the cosmological constant  $\Lambda$  (having equation of state parameter  $\omega_\Lambda = -1$ ) has been discarded both from theoretical as well as from observational point of view. There is a huge order of discrepancy in the observed value and the theoretically estimated value of  $\Lambda$  [2]-there are two well known difficulties namely the "fine tuning" and the "cosmic coincidence" problems and are commonly known as cosmological constant problems [3].

However, there are different candidate for the dynamical DE scenario in the literature to interpret the present accelerating phase of the universe namely a) the quintessence scalar field models [4], the phantom field [5], K-essence [6], tachyon field [7], quintom [8] etc. b) the DE models including Chaplygin gas [9], brane world models [10], holographic and agegraphic [12] models and so on.

From cosmological view point, it is interesting to consider interactions among the constituent matter components of the universe. But local gravity experiments put strong constraint on the interaction of DE with the baryonic matter [13] while there are no restrictions on the interaction among DE and DM; rather it is physically reasonable since DE gravitates - it may be accreted by massive compact objects (like BH, neutron star). But this energy flow from DE to DM should be small (but non-zero) from cosmological context.

Initially, the coupling between DE and DM was considered to reduce the huge difference between theoretically predicted value and the observed value [14] of the cosmological constant and to solve the coincidence problem [15]. Further it has been shown that a proper choice of the interaction term may influence the perturbation dynamics and affect the lowest

---

Department of Mathematics  
Jadavpur University  
Kolkata-700032  
E-mail: schakraborty@math.jdvu.ac.in · Department of Natural Science  
West Bengal University Of Technology  
Kolkata-700064  
E-mail: atreyee11@gmail.com

multipoles of the CMB spectrum [16, 17]. Also recently, the analysis of the supernova data together with CMB and large-scale structure [18] revealed such interaction from expansion history of the universe. Further, in the context of the dynamics of the galaxy clusters, signatures of the interaction between DE and DM has been analyzed [19].

Moreover, from thermodynamical view point the coupling between DE and DM has been studied [20] considering DE as perfect fluid with a well-defined temperature and it has been shown that at present epoch the energy flow should be from DE to DM for the validity of the second law of thermodynamics [21]. In the present work, we consider the universe containing interacting DE and DM as the matter constituent and it is assumed that universe bounded by the future event horizon is an isolated thermodynamical system. We shall consider thermodynamics of irreversible process (non equilibrium) due to energy flow and as a result the extensive property of the entropy of the whole system will no longer hold. We shall formulate the modified entropy of the whole system and examine the validity of the generalized second law of thermodynamics. The paper is organized as follows: section 2 deals with a general prescription for irreversible thermodynamics with DE as a perfect fluid with constant equation of state while holographic DE model has been used in section 3. At the end in section 4, there is a brief discussion and concluding remarks.

## 2 A study of the energy transfer between the dark sectors of the matter distribution: A general thermodynamic prescription

In the context of the recent observational evidences, the present day accelerating universe is dominated by an interacting two fluid system-the dark energy (DE) and dark matter (DM). The energy conservation relations for both the subsystems are

$$\dot{\rho}_m + 3H\rho_m = Q \quad (1)$$

and

$$\dot{\rho}_d + 3H(\rho_d + p_d) = -Q \quad (2)$$

where the DE component is a perfect fluid having energy density and thermodynamic pressure  $\rho_d$  and  $p_d$  respectively while DM is in the form of dust having energy density  $\rho_m$ . The interaction term  $Q > 0$  indicates an energy flow from DE to DM. The explicit form of  $Q$  is chosen in the form [22]

$$Q = 3H\lambda\rho_d \quad (3)$$

with  $\lambda$ , a small dimensionless positive quantity. Although this choice of  $Q$  seems to be phenomenological but it is compatible with observations [22, 23] like SNIa, CMB, large scale structure,  $H(z)$ , age constraints and recently in galaxy clusters. Also it has been shown to be effective in alleviating the coincidence problem.

As we are considering the fluid system in the universe, composed of two subsystems (DE+DM) at different temperatures interacting through exchange of energy, so it is reasonable to employ thermodynamics of irreversible process.

Accordingly, starting from the Euler's relation:  $nTs = \rho + p$ , ( $n$ =number density of particles in a comoving volume and  $s$ =the entropy per particle) and using the above conservation relations (1) and (2) and the conservation relation  $\frac{\dot{n}}{n} = -3H$  we have the evolution equations for temperature as

$$\frac{\dot{T}_m}{T_m} = 3H\frac{\lambda}{r} \quad (4)$$

and

$$\frac{\dot{T}_d}{T_d} = -3H(\lambda + \omega_d) \quad (5)$$

where  $T_m$  and  $T_d$  are the temperature of the DM component and the DE subsystem respectively,  $\omega_d$  ( $-1 < \omega_d < -\frac{1}{3}$ ) is the equation of state parameter for the DE and  $r (= \frac{\rho_m}{\rho_d})$  is the ratio of the energy densities of the two subsystems. Thus on integration we have

$$T_m = T_0 \left( \frac{r}{r_0} \right) \left( \frac{a}{a_0} \right)^{-\{2+3(\lambda+\omega_d)\}} \quad (6)$$

and

$$T_d = T_0 \left( \frac{a}{a_0} \right)^{-3(\lambda+\omega_d)} \quad (7)$$

where  $T_0$  is the common temperature of the two subsystems in equilibrium configuration while  $a_0, r_0$  are the values of the scale factor and the ratio of the energy densities in the equilibrium state. It is to be noted that in deriving equation (6) one has to take into account of the temperature  $T_{m_0} \propto a^{-2}$  for the DM sector in the absence of interaction. At very early stages of the evolution of the universe we have  $T_m > T_d$  and with the expansion of the universe, both the subsystems approach to the equilibrium configuration with common temperature  $T_0$  (when  $a = a_0$ ). Subsequently (i.e.  $a > a_0$ ), the thermal equilibrium is violated due to a continuous transfer of energy from DE to DM with  $T_m < T_0 < T_d$ . As we are considering the universe bounded by the event horizon as an isolated system, so at the thermal equilibrium the common temperature  $T_0$  is nothing but the Hawking temperature at the horizon i.e.  $T_0 = \frac{1}{2\pi R_E}$  where  $R_E$  is the radius of the event horizon for the FRW model.

For the present isolated system if we denote the entropies of the two subsystems as  $S_m$  and  $S_d$  and  $S_E$  is the entropy of the bounding event horizon, then

$$T_m \frac{dS_m}{dt} = \frac{dQ_m}{dt} = \frac{dE_m}{dt} \quad (8)$$

and

$$T_d \frac{dS_d}{dt} = \frac{dQ_d}{dt} = \frac{dE_d}{dt} + p_d \frac{dV}{dt} \quad (9)$$

while from the Bekenstein area formula,

$$\frac{dS_E}{dt} = 2\pi R_E \dot{R}_E \quad (10)$$

Here  $V = \frac{4}{3}\pi R_E^3$  is the volume of the universe bounded by the event horizon and  $E_m = \rho_m V$  and  $E_d = \rho_d V$ .

As the overall system is isolated so the heat flow across the horizon ( $Q_h$ ) will satisfy

$$\dot{Q}_h = -(\dot{Q}_m + \dot{Q}_d) \quad (11)$$

In equilibrium configuration, the entropy of the whole system depends on the energy densities and volume only and from the extensive property, it is just the sum of the entropies i.e.  $S_m + S_d + S_E$ . However in non-equilibrium thermodynamics one has to take into account of the irreversible fluxes such as energy transfers in the total entropy and hence the time variation of the total entropy is given by

$$\frac{dS_T}{dt} = \frac{dS_m}{dt} + \frac{dS_d}{dt} + \frac{dS_E}{dt} - A_d \dot{Q}_d \ddot{Q}_d - A_h \dot{Q}_h \ddot{Q}_h \quad (12)$$

where  $A_d$  and  $A_h$  are the energy transfer constants between DE and DM within the universe and between the universe and the horizon respectively. Now using equations (8)-(10) the explicit form of different terms on the r.h.s of equation (12) are given by

$$\begin{aligned} \frac{dS_m}{dt} &= -3\pi H^2 R_E^3 \left( \frac{1+z_{eq}}{1+z} \right)^{2+3(\lambda+\omega_d)} \times \\ &\quad \left( \frac{r_{eq}}{r} \right) \{1 + \Omega_k - \Omega_d(1 + \lambda H R_E)\} \\ \frac{dS_d}{dt} &= -3\pi H^2 R_E^3 \left( \frac{1+z_{eq}}{1+z} \right)^{3(\lambda+\omega_d)} \Omega_d \times \\ &\quad (1 + \omega_d + \lambda H R_E) \\ \frac{dS_E}{dt} &= 2\pi R_E (H R_E - 1) \\ A_d \dot{Q}_d \ddot{Q}_d &= -\frac{9}{4} A_d H^4 R_E^3 (1 + \omega_d + \lambda H R_E) [2(1 + \omega_d) \\ &\quad + H R_E \{3(1 + \omega_d)^2 + (3\lambda - 2)(1 + \omega_d) + 3\lambda\} \\ &\quad + H^2 R_E^2 \{3\lambda^2 + \lambda(3\omega_d + q + 1)\}] \\ A_h \dot{Q}_h \ddot{Q}_h &= -\frac{9}{4} A_d H^4 R_E^3 (1 + \Omega_k + \omega_d \Omega_d) [2(1 + \Omega_k \\ &\quad + \omega_d \Omega_d) + 2q H R_E + \omega_d \Omega_d H R_E \{3(1 + \omega_d) \\ &\quad + 3\lambda - 2\}] \end{aligned}$$

with  $z$ , the usual red-shift parameter.

As the expression for  $\frac{dS_T}{dt}$  is very lengthy, so to get an idea about its sign we make use of the observed or estimated values of different parameters present in the above expressions at present epoch as follows:

$$\omega_d = -1, \lambda = \frac{1}{3}, z_{eq} = 5.56 \times 10^7, r_{eq} = 1.09 \times 10^5, \Omega_d = 0.72, \Omega_k = 0.02; z = 0, q = -0.57.$$

So we have

$$\begin{aligned} \frac{dS_T}{dt} &= R_E [5.9 \times 10^5 (H R_E)^2 (H R_E - 1.25) + \\ &\quad 0.2 \bar{A}_d (H R_E)^4 (1.92 - H R_E) + 0.28 \bar{A}_h (H R_E)^2 \times \\ &\quad (1.43 - H R_E) + 6.28 (H R_E - 1)] \end{aligned}$$

where  $\bar{A}_d = A_d H^2$  and  $\bar{A}_h = A_h H^2$

So if we take  $\bar{A}_d, \bar{A}_h > 0$  and  $1.25 R_A < R_E < 1.43 R_A$  we see that  $\frac{dS_T}{dt} > 0$  i.e generalized second law of thermodynamics (GSLT) holds on the event horizon for the present irreversible thermodynamical system.

### 3 Holographic Dark Energy Model

A typical dark energy model which satisfies the holographic principle is known as holographic dark energy (HDE) model. According to this model using effective quantum field theory the energy density is given by [11]

$$\rho_d = \frac{3c^2}{R_E^2}$$

where 'c' is a dimensionless parameter which may be estimated from observation [24] and the radius of the event horizon is chosen as the IR cut-off length to obtain correct equation of state and the desired accelerating universe [25]. So one can write  $R_E$  as

$$R_E = \frac{c}{\sqrt{\Omega_d H}}$$

where  $\Omega_d = \frac{8\pi\rho_d}{3H^2}$  is the density parameter.

At first for simplicity of calculations we use the dark sector as the non-interacting two subsystems namely the HDE and the DM. Then the density parameter evolves as [16]

$$\Omega'_d = \Omega_d (1 - \Omega_d) \left( 1 + \frac{2\sqrt{\Omega_d}}{c} \right) \quad (13)$$

and the variable equation of state parameter for the HDE is

$$\omega_d = -\frac{1}{3} - \frac{2\sqrt{\Omega_d}}{3c} \quad (14)$$

where 'x' stands for differentiation with respect to  $x = \ln a$ .

In thermodynamics, starting from Euler's relation, the temperature of the HDE (a perfect fluid with variable equation of state) can be written as

$$T_d = T_{d0} (1 + \omega_d) e^{-3 \int \omega_d dx} \quad (15)$$

which on integration using (13) gives

$$\begin{aligned} T_d &= T_{d0} \frac{a}{a_0} \left( 1 - \frac{\Omega_d}{c} \right) (1 - \sqrt{\Omega_d})^{\frac{2}{c+2}} (1 + \sqrt{\Omega_d})^{\frac{2}{c-2}} \\ &\quad \times \left( 1 + \frac{2}{c} \sqrt{\Omega_d} \right)^{\frac{8}{4-c^2}} \end{aligned} \quad (16)$$

with  $T_{d0}$  an integration constant and  $a_0$  the value of 'a' when DE and DM are in thermal equilibrium. Also the temperature of the DM subsystem (behaving as dust) varies as the reciprocal of the square of the scale factor [[22]] i.e

$$T_m \propto T_{m0} a^{-2} \quad (17)$$

$$\text{or, } T_m = T_{m0} \left( \frac{a}{a_0} \right)^{-2} \quad (18)$$

Then by virtue of the extensive property, the entropy of the whole system is just the sum of the entropies of the subsystems and the entropy of the horizon, i.e.,

$$\frac{dS_T}{dt} = \frac{dS_m}{dt} + \frac{dS_d}{dt} + \frac{dS_E}{dt}$$

Then as before using Gibbs' law to obtain the explicit form of the first two terms on the r.h.s of above equation and using Bekenstein entropy-area formula for the 3rd term we obtain

$$\begin{aligned} \frac{dS_T}{dt} = 2\pi R_E \left[ HR_E - 1 - \frac{2}{T_{m0}} \frac{R_E \rho_m (1+z_{eq})^2}{(1+z)^2} - \right. \\ \left. \frac{4R_E \rho_d (1+z)}{3T_{d0}(1+z_{eq})} (1-\sqrt{\Omega_d})^{\frac{2}{c+2}} (1+\sqrt{\Omega_d})^{\frac{2}{c-2}} \right. \\ \left. \left(1 + \frac{2\sqrt{\Omega_d}}{c}\right)^{-\frac{8}{4-c^2}} \right] \end{aligned}$$

Now using  $T_{m0} = T_{d0} = \frac{1}{2\pi R_E}$ , the Hawking temperature associated with event horizon when DE and DM are in equilibrium, we obtain

$$\frac{dS_T}{dt} = 2\pi R_E (x - lx^2)$$

$$\begin{aligned} \text{where } x = HR_E, l = 4\pi \left[ 3(1-\Omega_d)a^2 + \frac{2\Omega_d b}{a} \right], a = \frac{1+z_{eq}}{1+z}, \\ b = (1-\sqrt{\Omega_d})^{\frac{2}{c+2}} (1+\sqrt{\Omega_d})^{\frac{2}{c-2}} \left(1 + \frac{2\sqrt{\Omega_d}}{c}\right)^{-\frac{8}{4-c^2}} \end{aligned}$$

Hence for the validity of GSLT we have  $l < \frac{1}{4}$  and  $\alpha R_A < R_E < \beta R_A$  with  $\alpha, \beta = \left(\frac{1 \pm \sqrt{1-4l}}{2l}\right)$

Now we shall generalize our model by considering interaction between HDE and DM. The form of the interaction term is chosen same as in the previous section. Then the evolution of the density parameter and the equation of state parameter for HDE are modified as

$$\Omega'_d = \Omega_d(1-\Omega_d) \left(1 + \frac{2\sqrt{\Omega_d}}{c}\right) - 3\lambda \Omega_d^2$$

and

$$\omega_d = -\left(\lambda + \frac{1}{3}\right) - \frac{2\sqrt{\Omega_d}}{3c}$$

Now integrating the energy conservation equations the explicit form of the energy density components are obtained as:

$$\rho_m = \rho_{m0} \left(\frac{a}{a_0}\right)^{-3} \exp \left[ 3\lambda \int \frac{da}{ar} \right]$$

and

$$\rho_d = \rho_{d0} \left(\frac{a}{a_0}\right)^{-2} \exp \left[ \frac{4}{c} \int \frac{dz}{(1-z^2)(1+\frac{2z}{c}) - 3\lambda z^2} \right]$$

In non equilibrium extended thermodynamics due to irreversible fluxes like energy transfers the entropy of the whole

system does not satisfy the extensive property (as in equilibrium case), rather it modifies as equation (12).

The temperature of the two dark sectors are now given by

$$T_d = T_{d0} \left( \frac{2}{3} - \lambda - \frac{2\sqrt{\Omega_d}}{3c} \right) \frac{a}{a_0} I$$

and

$$T_m = T_{m0} \frac{r}{r_0} \frac{a}{a_0} I$$

where  $r = \frac{\rho_m}{\rho_d}, r_0 = \frac{\rho_{m0}}{\rho_{d0}}, z = \sqrt{\Omega_d}$  and  $T_{m0}$  and  $T_{d0}$  are integration constants and  $I = \exp \left[ \frac{4}{c} \int \frac{dz}{(1-z^2)(1+\frac{2z}{c}) - 3\lambda z^2} \right]$

However if we restrict ourselves to flat universe, then

$$\frac{\dot{Q}_m}{T_m} = \frac{3\pi c^3 \sqrt{\Omega_d} r_0 (1+z) H^{-1}}{(1-\Omega_d)(1+z_{eq})I} \left[ 1 + \frac{\lambda c}{\sqrt{\Omega_d}} - \frac{1}{\Omega_d} \right]$$

$$\frac{\dot{Q}_d}{T_d} = \frac{3\pi c^3 (1+z) H^{-1}}{\sqrt{\Omega_d} \left( \frac{2}{3} - \lambda - \frac{2\sqrt{\Omega_d}}{3c} \right) (1+z_{eq})I} \left[ \frac{2}{3c} \sqrt{\Omega_d} - \frac{\lambda c}{\sqrt{\Omega_d}} + \lambda - \frac{2}{3} \right]$$

$$\dot{Q}_d \ddot{Q}_d = \frac{\Omega'_d}{\Omega_d^2} \frac{9c^4 H}{4} \left[ \frac{2\Omega_d^2}{9c^2} + \left( \lambda - \frac{2}{3} \right) \frac{\Omega_d^{\frac{3}{2}}}{3c} + \frac{\lambda c \sqrt{\Omega_d} (\lambda - \frac{2}{3})}{2} - \frac{\lambda^2 c^2}{2} \right]$$

$$\dot{Q}_h \ddot{Q}_h = \frac{\Omega'_d}{\Omega_d^2} \frac{9c^4 H}{4} \left[ \frac{2\Omega_d^2}{9c^2} + \left( \lambda + \frac{1}{3} \right) \frac{\Omega_d^{\frac{3}{2}}}{3c} + \frac{\sqrt{\Omega_d}}{3c} - \frac{1}{\Omega_d} + \lambda + \frac{1}{3} \right]$$

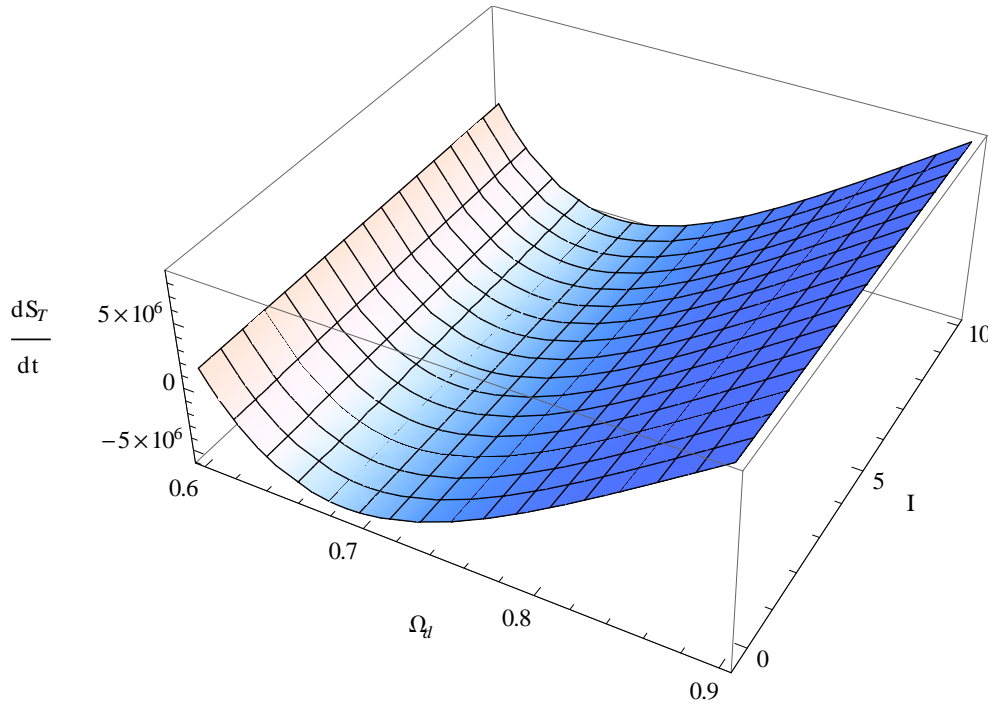
$$\frac{dS_E}{dt} = 2\pi R_E \dot{R}_E = 6.28cH^{-1} \left( \frac{c}{\Omega_d} - \frac{1}{\sqrt{\Omega_d}} \right)$$

So the time variation of total entropy reads as

$$\begin{aligned} \frac{dS_T}{dt} = \frac{3\pi c^3 H^{-1} I^{-1} (1+z)}{1+z_{eq}} \left[ \frac{r_0 \sqrt{\Omega_d}}{1-\Omega_d} \left( 1 + \frac{\lambda c}{\sqrt{\Omega_d}} - \frac{1}{\Omega_d} \right) \right. \\ \left. + \frac{\lambda - \frac{2}{3} + \frac{2\sqrt{\Omega_d}}{c} - \frac{\lambda c}{\sqrt{\Omega_d}}}{\sqrt{\Omega_d} \left( \frac{2}{3} - \lambda - \frac{2\sqrt{\Omega_d}}{c} \right)} \right] + \frac{9c^4 \Omega'_d H^{-1}}{4\Omega_d^2} \left[ \bar{A}_d \left( -\frac{2\Omega_d^2}{9c^2} \right. \right. \\ \left. \left. - \frac{(\lambda - \frac{2}{3})}{9c} \Omega_d^{3/2} - \frac{\lambda c (\lambda - \frac{2}{3})}{2} \sqrt{\Omega_d} + \frac{\lambda^2 c^2}{2} \right) + \right. \\ \left. \bar{A}_h \left( -\frac{2\Omega_d^2}{9c^2} - \frac{(\lambda + \frac{1}{3})}{9c} \Omega_d^{3/2} - \frac{\sqrt{\Omega_d}}{3c} + \frac{1}{\Omega_d} - \lambda - \frac{1}{3} \right) \right] \\ + 6.28cH^{-1} \left( \frac{c}{\Omega_d} - \frac{1}{\sqrt{\Omega_d}} \right) \end{aligned}$$

with  $\bar{A} = AH^2$

From the above expression for  $\frac{dS_T}{dt}$  it is not possible to examine the validity of GSLT, however we give a graphical representation of  $\frac{dS_T}{dt}$  with the variation of  $\Omega_d$  and  $I$  in figure 1. The figure tells us that with the observed dominance of dark energy (i.e.  $\Omega_d \geq 0.75$ ) GSLT is always satisfied.



**Fig. 1** Graphical representation of  $\frac{dS_T}{dt}$  in the unit of  $H^{-1}$  with the variation of  $\Omega_d$  and  $I$  for  $\lambda = \frac{1}{3}$ ,  $z_{eq} = 5.56 \times 10^7$ ,  $r_{eq} = 1.09 \times 10^5$ ,  $\bar{A}_d = \bar{A}_h = 3.56 \times 10^7$ ,  $z = 0$

#### 4 Discussion and Concluding remarks:

A study of non-equilibrium thermodynamics for the universe bounded by the event horizon has been done with matter content as interacting two-fluid system - the two dark components known at present as dark matter and dark energy. As usual, the dark matter is chosen in the form of dust while in two separate sections the dark energy is chosen as perfect fluid with constant equation of state and holographic dark energy model respectively. Irreversible thermodynamics is applied to the isolated system (i.e. universe bounded by the event horizon) as the mutual interaction between the two dark fluid species results in a spontaneous heat flow between the horizon and the fluid system. At early epoch of the evolution of the universe the temperature of the DM is larger than that of DE and both approaches the Hawking temperature of the horizon in course of expansion of the universe. However, subsequently this equilibrium configuration is destroyed due to a continuous transfer of energy from DE to DM, and hence the extensive property of the entropy of the whole system can not be applicable to the present system. Though the expression for the time variation of the total entropy of the system is very complicated but it is possible to find restrictions for the validity of the GSLT in case of perfect fluid model of DE with present observed data and of holographic dark energy model (without interaction). In both cases radius of the event horizon is restricted in a range for which both the bounds are proportional to the radius

of the apparent horizon. On the other hand, in case of holographic dark energy with interaction, even the temperature can only be evaluated in integral form and hence no explicit analytic form for total entropy variation is possible. So analytically we do not have any conclusion regarding validity of GSLT, however, we have shown only graphically. Finally we make a comparison with previous similar work done by Karami et al [26] for universe bounded by apparent horizon. In that work they have only considered DE as perfect fluid with constant equation of state and have shown the validity of the GSLT with a restriction using the present observed data. In the present work, our derivation of non-equilibrium temperature of the two dark sectors with variable equation of state is new and also use of event horizon for interacting dark fluid is initiated in the present work. For future work, we shall try with other dark energy models for non-equilibrium thermodynamics.

**Acknowledgements** The authors are thankful to IUCAA for research facilities as a part of the work is done here.

#### References

1. A.G. Riess et al, *Astron. J.*, 116, 1009 (1998);  
S. Perlmutter et al, *Astrophysics. J.*, 517, 565 (1999);  
P. de Bernardis et al, *Nature*, 404, 955 (2000);  
S. Perlmutter et al., *Astrophysics. J.*, 598, 102 (2003)



2. S. Weinberg, *Rev. Mod. Phys.* 61, 1 (1989)
3. E. J. Copeland, M. Sami, S. Tsujikawa, *Int. J. Mod. Phys.*, D15, 1753 (2006)
4. C. Wetterich, *Nucl. Phys. B* 302, 668 (1988); B. Ratra, J. Peebles, *Phys. Rev. D* 37, 321 (1988)
5. R. R. Caldwell, *Phys. Lett. B* 545, 23 (2002); S. Nojiri, S. D. Odinstov, *Phys.* 562, 147 (2003); *Phys. Lett. B* 565, 1 (2003)
6. T. Chiba, T. Okabe, M. Yamaguchi, *Phys. Rev. D* 62, 023511 (2000); C. Armendáriz-Picón, M. Mukhanov P. J. Steinhardt, *Phys. Rev. Lett.* 85, 4438 (2000); *Phys. Rev. D* 63, 103510 (2001)
7. A. Sen, *J. High Energy Phys.*, 04, 048 (2002); T. Padmanabhan, T. R. Chaudhury, *Phys. Rev. D* 66, 081301 (2002)
8. E. Elizalde, S. Nojiri, S. D. Odinstov, S. Tsujikawa, *Phys. Rev. D* 71, 063004 (2005); A. Anisimov, E. Bubichev, A. Vikman, *J. Cosmol. Astropart. Phys.*, 06, 006 (2005)
9. A. Kamenshchik, U. Maschella, V. Pasquier, *Phys. Lett. B* 511, 265 (2001); M. C. Bento, O. Bertolami, A. A. Sen, *Phys. Rev. D* 66, 043507 (2002)
10. C. Deffayet, G. R. Dvali, G. Gabadadze, *Phys. Rev. D*, 65, 044023 (2002); V. Sahni, Y. Shtanov, *J. Cosmol. Astropart. Phys.*, 11, 014 (2003)
11. A. Cohen, D. Kaplan, A. Nelson, *Phys. Rev. Lett.* 82, 4971 (1999); P. Horava, D. Minic, *Phys. Rev. Lett.* 85, 1610 (2000); S. D. Thomas, *Phys. Rev. Lett.* 89, 081301 (2002); M. Li, *Phys. Lett. B*, 603, 1 (2004)
12. R. G. Cai, *Phys. Lett. B* 657, 228 (2007); H. Wei, R. G. Cai, *Phys. Lett. B*, 660, 113 (2008); *Phys. Lett. B* 663, 1 (2008); *Eur. Phys. J. C* 59, 99 (2009); K. Y. Kim, H. W. Lee, Y. S. Myung, *Phys. Lett. B* 660, 118 (2008); J. Zhang, X. Zhang, H. Liu, *Eur. Phys. J. C* 54, 303 (2008)
13. P. J. E. Peebles, B. Ratra, *Rev. Mod. Phys.* 75, 559 (2003); K. Hagiwara et al; *Phys. Rev. D* 66, 010001 (2002)
14. C. Wetterich, *Nucl. Phys. B* 302, 668 (1988)
15. L. Amendola, S. Tsujikawa, M. Sami, *Phys. Lett. B* 632, 155 (2006); W. Zimdahl, D. Pavón, *Phys. Rev. D* 70, 043540 (2004); G. Olivares, F. Atrio-Barandela, D. Pavón, *Phys. Rev. D* 74, 043521 (2006); *Phys. Rev. D* 78, 021302 (R) (2008)
16. B. Wang, Y. G. Gong, E. Abdalla, *Phys. Lett. B* 624, 141 (2005);
17. B. Wang, J. D. Zang, C. -Y. Lin, E. Abdalla, S. Micheletti, *Nucl. Phys. B* 778, 69 (2007)
18. C. Feng, B. Wang, Y. G. Gong, R. -K. Su, *J. Cosmol. Astropart. Phys.* 09, 005 (2007); Z. K. Guo, N. Ohta, S. Tsujikawa, *Phys. Rev. D* 76, 023508 (2007); J. H. He, B. Wang, *J. Cosmol. Astropart. Phys.* 06, 010 (2008); C. Feng, B. Wang, E. Abdalla, R. -K. Su, *Phys. Lett. B* 665, 111 (2008)
19. O. Bertolami, F. Gil Pedro, M. Le Delliou, *Phys. Lett. B* 654, 165 (2007); E. Abdalla, L. Raul, W. Abramo, L. Sodre, Jr., B. Wang, *arxiv* 0710.1198 (astro-ph)
20. B. Wang, C. Y. Lin, D. Pavón, E. Abdalla, *Phys. Lett. B* 662, 19 (2008); D. Pavón, B. Wang, *arXiv*:0912.0565
21. H. Callen, *Thermodynamics* (J. Wiley, 1960)
22. D. Pavón, W. Zimdahl, *Phys. Lett. B* 628, 206 (2005); *class Quantum Grav.* 24, 5461 (2007); J. -H. He, B. Wang, *J. Cosmol. Astropart. Phys.* 06, 010 (2008)
23. A. A. Sen, D. Pavón, *Phys. Lett. B* 664, 7 (2008)
24. Q. G. Huang, M. Li, *JCAP* 0408, 013 (2004); Z. Chang, F. -Q. Wu, X. Zhang, *Phys. Lett. B* 633, 14 (2006); Z. Chang, F. -Q. Wu, *Phys. Rev. D* 72, 043524 (2005); *Phys. Rev. D* 76, 023502 (2007); E. N. Saridakis, M. R. Setare, *Phys. Lett. B* 670, 01 (2008)
25. M. Li, *Phys. Lett. B* 603, 01 (2004)
26. K. Karami, S. Ghaffari, *Phys. Lett. B* 685, 115-119 (2010)